Quantitative Products Analytics

Deutsche Bank's Equity Derivatives Quant Team

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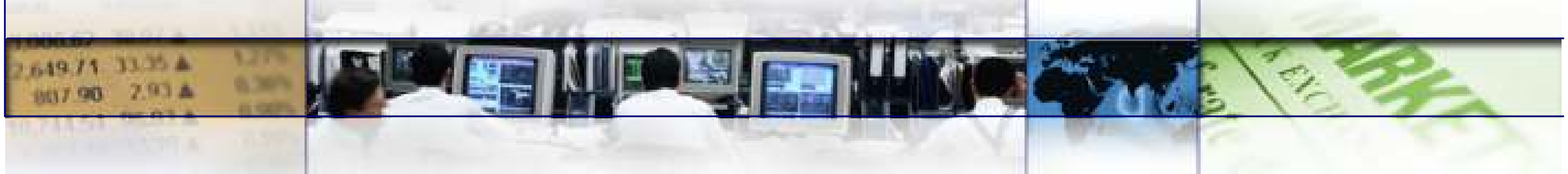
University Paris 6, Friday,
January 26th, 2007

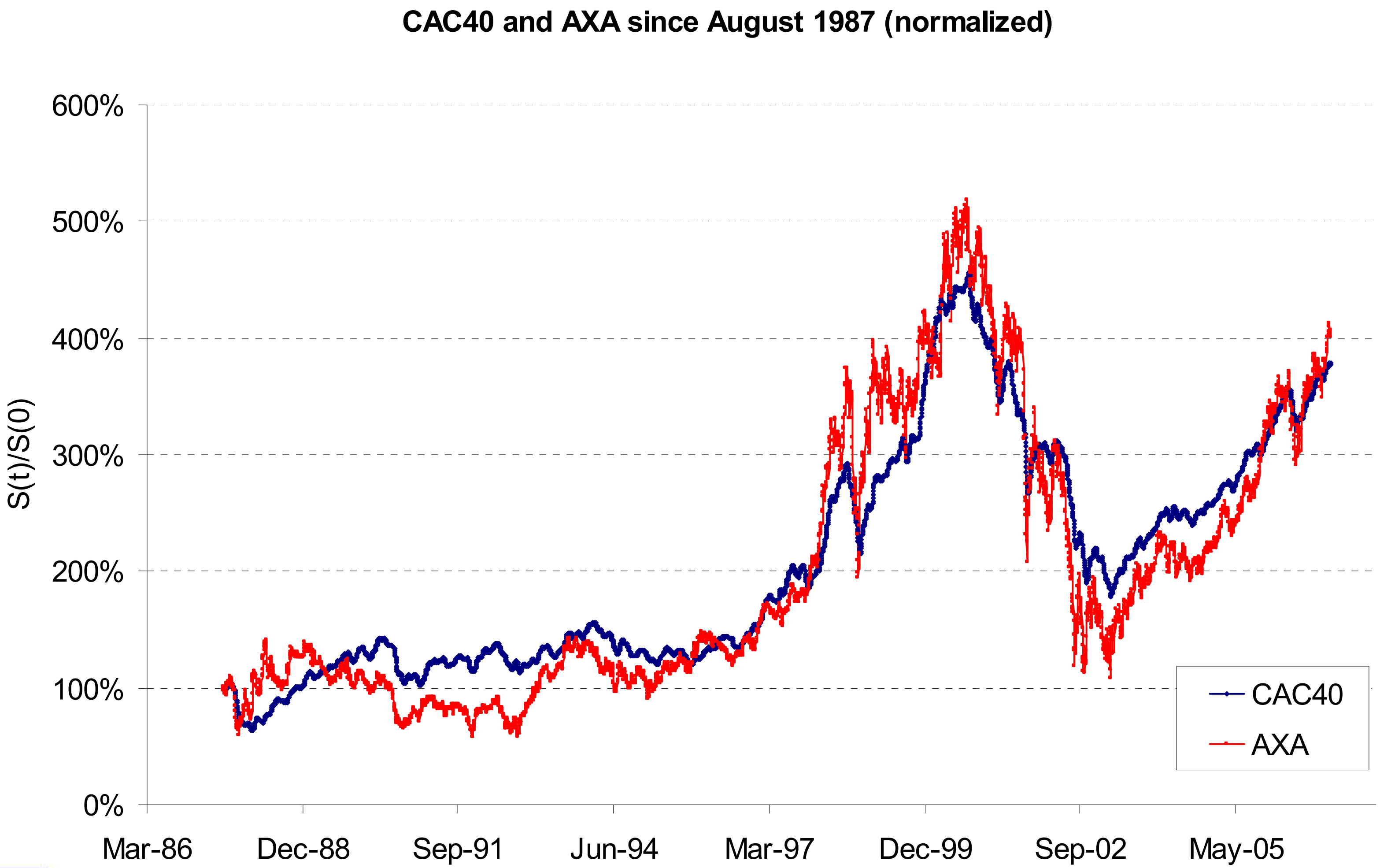


DB Quantitative Products Analytics

The team

- Head of all Quantitative Products (Quants, Engineering, Algorithmic Trading Quants): Dr.^{2} Marcus Overhaus
- „Analytics“ quant team consists currently of 10 members, to grow to 13 by end of March
  - 12 in London
  - 1 senior US quant
- Qualifications and nationalities widely mixed
- Opened research institute *„QP Laboratory“* in cooperation with two Berlin universities (three professors with PhD and PostDoc students)
  - Research on fundamental questions
  - PhD projects together with the quant team

Equity Derivatives



Equity Derivatives

Examples

- European/American vanilla options on single names/indices.
  - Often part of a packaged structure, for example a „discount certificate“:

$$\min\left\{\frac{S_T}{S_0}, 120\%\right\} = \frac{S_T}{S_0} - \left(\frac{S_T}{S_0} - 120\%\right)^+$$

The short call makes this product cheaper than a straight investment in the equity.

- Barriers such as down-and-in puts

$$\left(100\% - \frac{S_T}{S_0}\right)^+ 1_{\inf S_T < 70\% S_0}$$

A cheap way to participate strongly in a severe downturn.

Equity Derivatives
Examples

- Cliquets
  - E.g. three-monthly capped and floored performance of an equity with capital protection.

$$\left(\sum_{i=1,\dots,n} \max\left\{ -5\%, \min\left\{ +5\%, \frac{S_{t_i}}{S_{t_{i-1}}} - 1 \right\} \right\} \right)^{+}$$

Each element of the inner sum is a three-monthly performance, limited to the range [-5%,+5%]

- Hybrid-trades such knock-out coupon streams:
  - The client receives a large coupon until an upper barrier is hit.

$$\sum_{i=1,\dots,n} \left(\text{Libor}(t_i) + 2\% \right) 1_{\sup S_{t_i} < S_0 110\%}$$

Paricularly popular in Japan.

Equity Derivatives
Clients

- Retail (third party or our own networks in Germany, Poland, Spain)
  - Capital protection
  - Discount certificates
  - High-coupon conditional on stock („express certificates“)
- Corporate clients
  - Capital protected investment
  - Hedging
  - Diversification (volatility trades, cross-asset baskets, ...)
- Global Players (Hedge-Funds etc)
  - Speculation
  - Statistical Arbitrage
  - Hedging

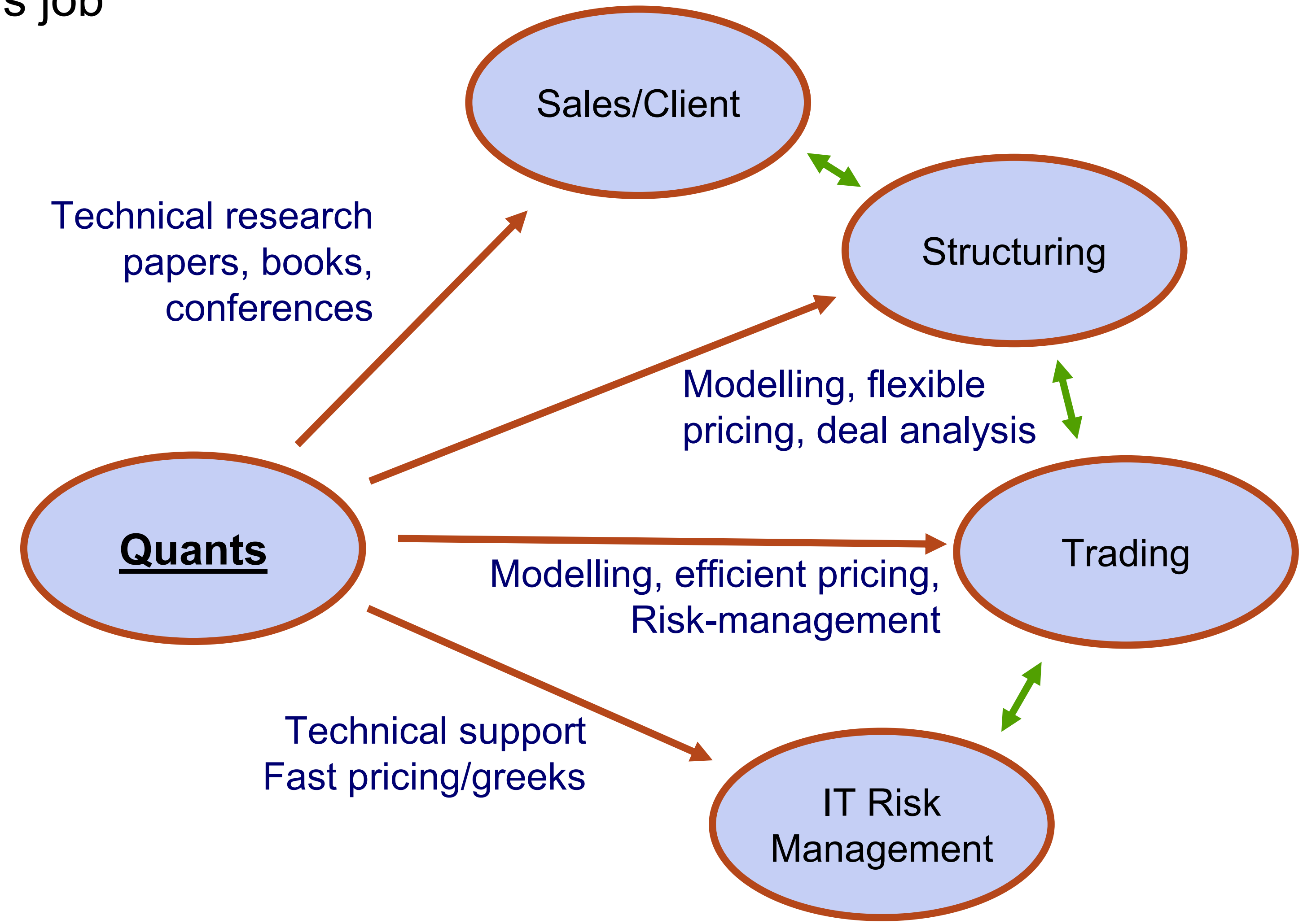
Equity Derivatives

Where are new products coming from?

- Innovation
  - From trading: book position favours certain trades to „unload risk“
  - From sales: understanding client‘s needs and intentions
  - From structuring: certain trades make sense (economical, statistical, ...)
  - From quants: availability of hedges
- Client initiative
- Competition

DB Equity Derivatives

A quant's job



Equity Derivatives
Recent modelling challenges

- Options on correlation
  - Correlation swaps trade the average correlation of a basket
  - Options on correlation are the next logical step
- Options on dividends
  - Dividend swaps allows to lock-in future dividends of an asset
  - Options on dividends allow to ensure minimum levels of cash flows
- Options on variance ....

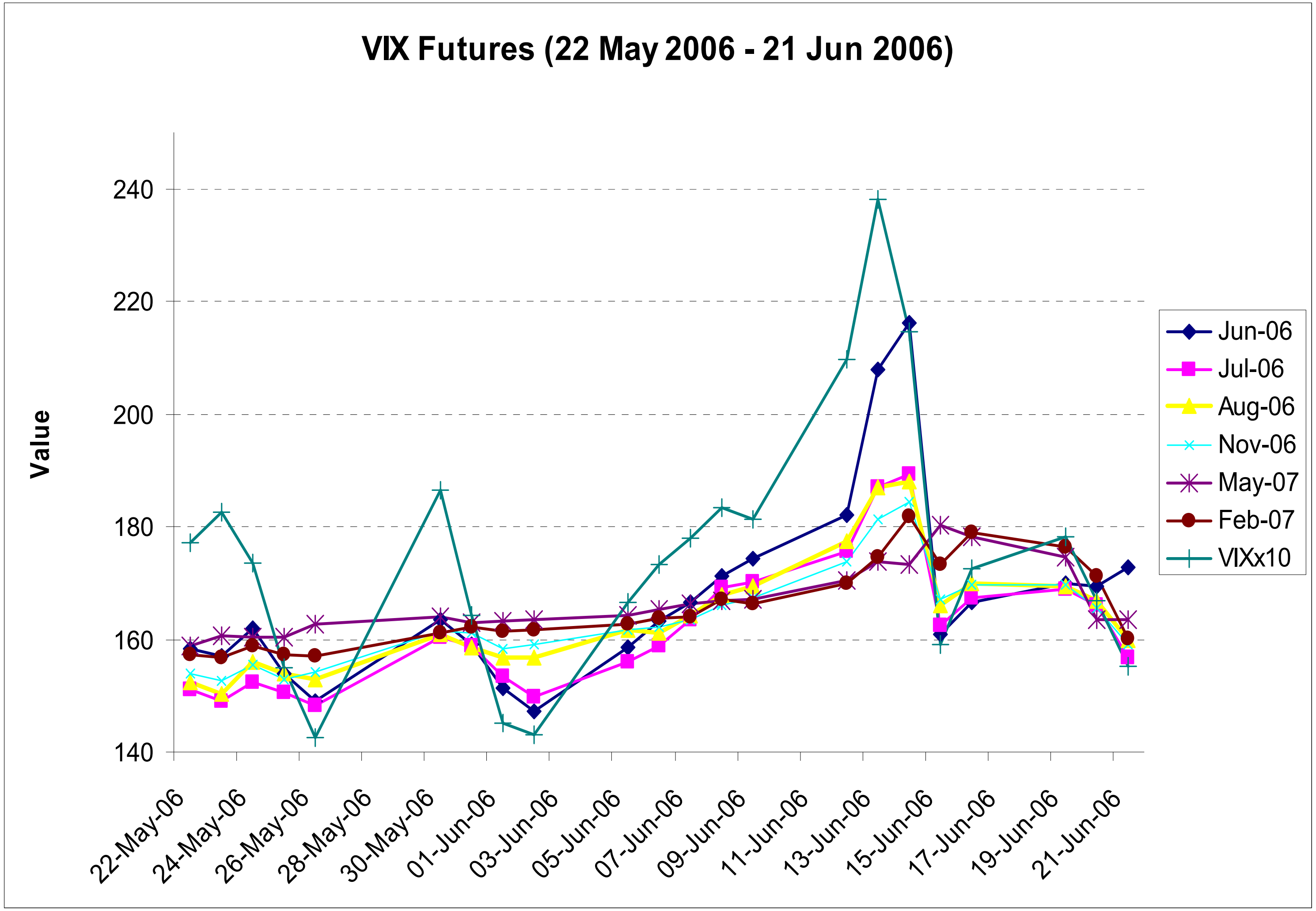
Options on Variance

Outline

- Trading volatility
  - Variance Swaps
  - Options On Realized Variance
- Model specification
  - Heston example
  - Fitting automatically Variance swap curve
- Fitting Fitted Heston
- Sensitivities with respect to the parameters of the model
- Implied Volatility of Options on Variance

Trading volatility

Trading volatility



Variance swaps

- The payoff of the **Variance Swap** is Realized Variance – (Strike)^2 :

$$\frac{252}{n-1}\sum_{k=1}^{n}\left(\log\frac{S_{t_k}}{S_{t_{k-1}}}\right)^2 - K^2$$

- Why Variance Swaps?
  - Allows taking position in volatility. Vega is much less dependent on a spot level than it is for vanilla structures
  - Diversifying portfolio. Moves in volatility and underlying are negatively correlated
  - More or less Model independent pricing. Can be statically replicated with vanilla options

Variance swaps

- Suppose that pure stock price is a continuous martingale, interest rates and dividends are zero, i.e.

$$dS_t = \sigma_t S_t dW_t$$

- From Ito's formula we get:

$$\log(S_T) = \log(S_0) + \int_0^T \sigma_s dW_s - \frac{1}{2}\int_0^T \sigma_u^2 du$$

- We also have

$$\frac{1}{T}\sum_{k=1}^{n}\left(\log\frac{S_{t_k}}{S_{t_{k-1}}}\right)^2 \approx \frac{1}{T}\langle \log S, \log S\rangle_T$$

Variance swaps (continued)

- Hence we obtain that

$$\langle \log S, \log S \rangle_T = \int_0^T \sigma_s^2 ds$$

- From the same Ito's formula we get

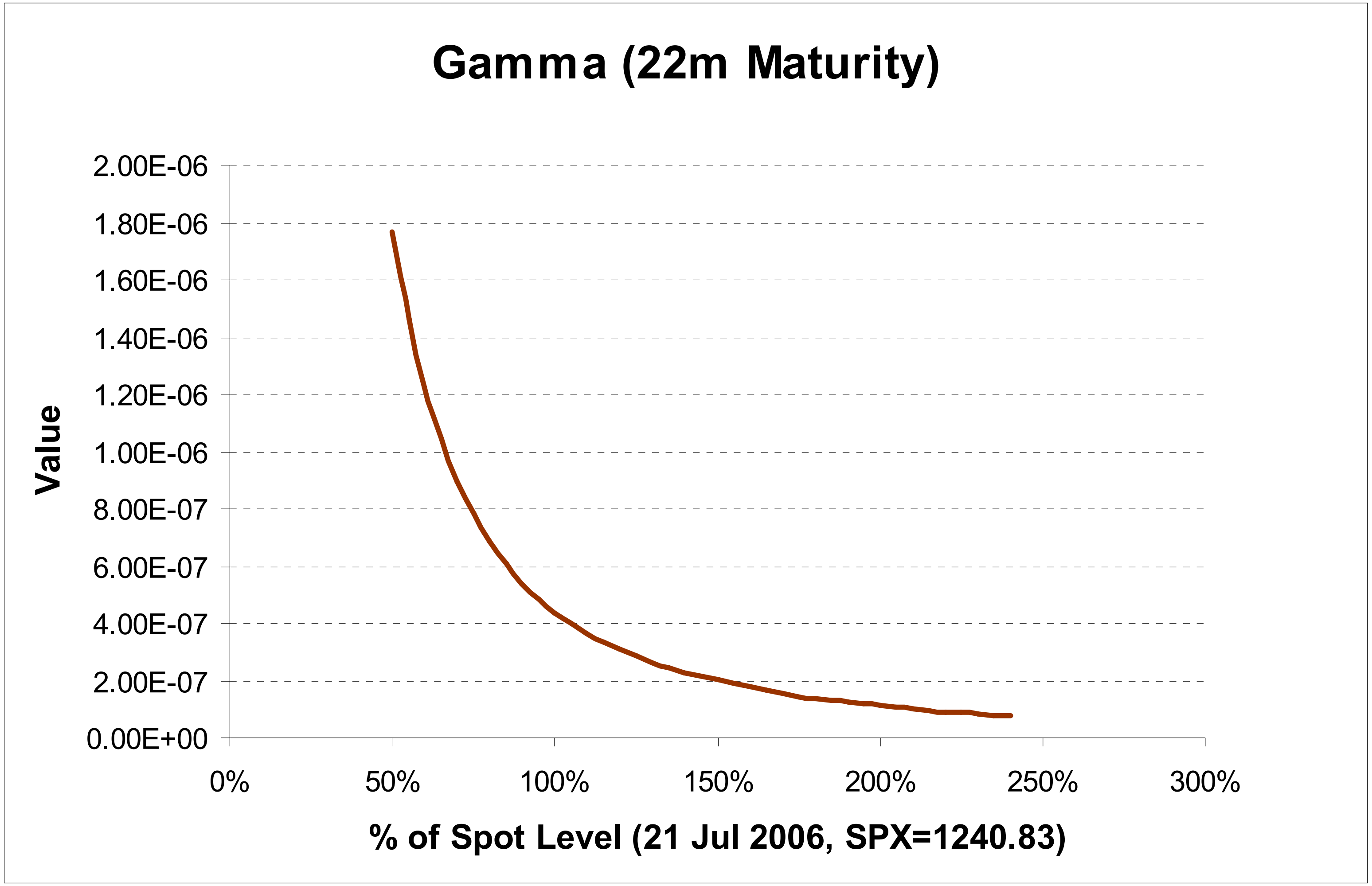
$$\int_0^T \sigma_s^2 ds = -2\log(S_T) + 2\log(S_0) + 2\int_0^T \frac{1}{S_u} dS_u$$

- Finally using risk neutral valuation we get

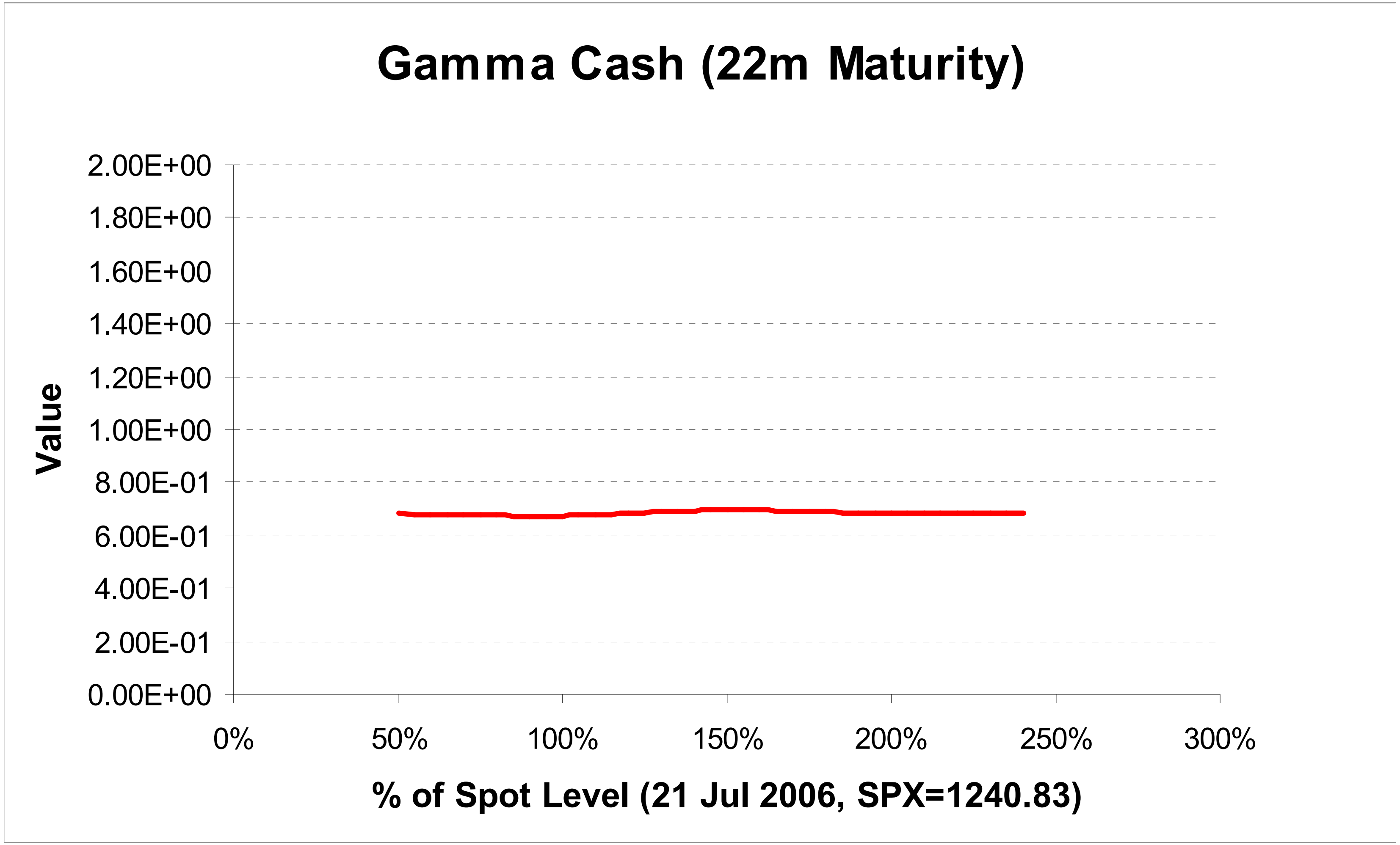
$$V_0(T) = E\int_0^T \sigma_s^2 ds = -2E\log\left(\frac{S_T}{S_0}\right)$$

- We can statically hedge the log payoff with Puts and Calls

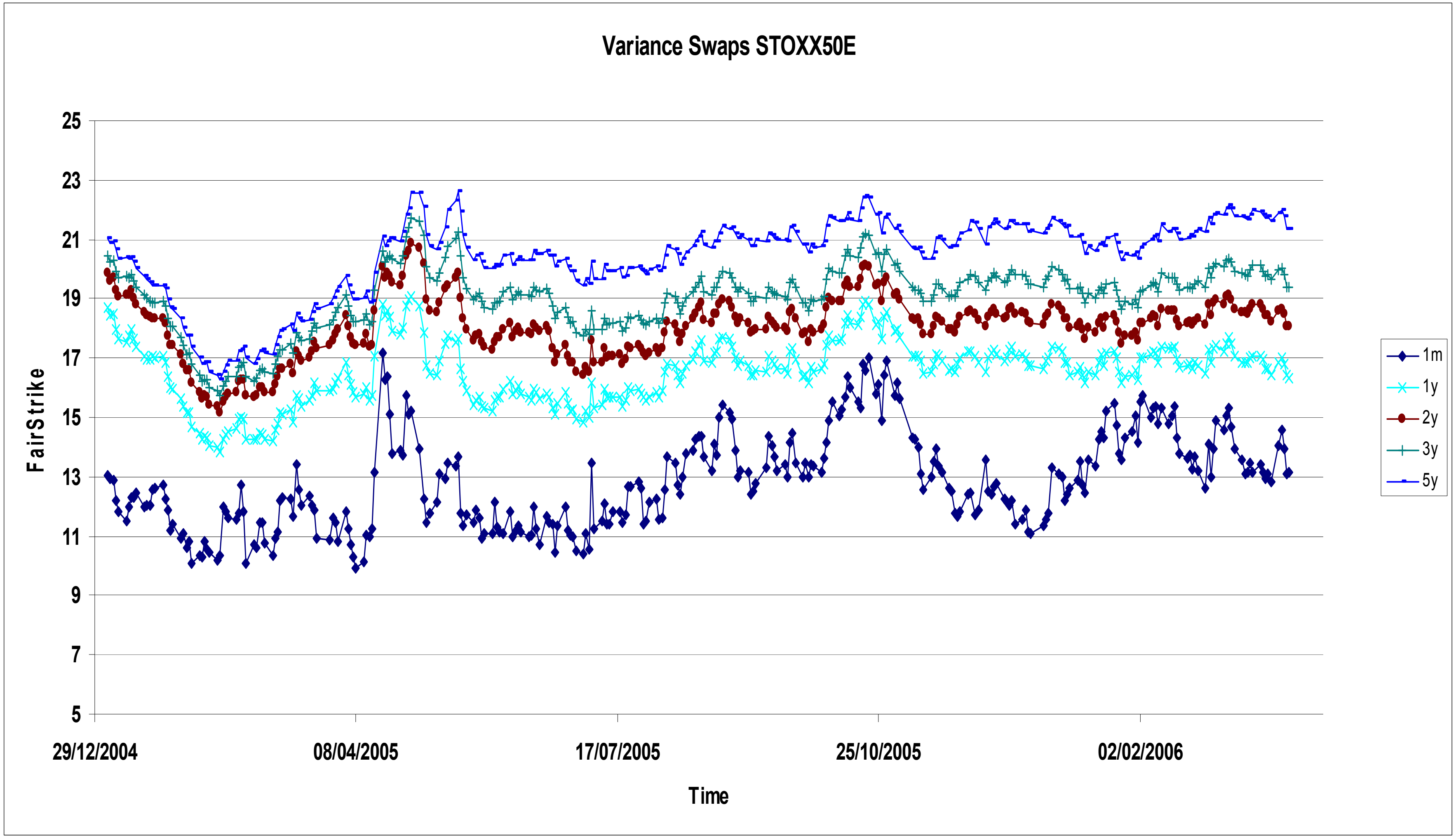
Gamma of a log payoff



Gamma of a log payoff



Variance Swaps



Options on Realized Variance

- The payoff of **Option on Realized Variance** is

$$\left(\frac{252}{n-1}\sum_{i=1}^{n}\log^2\left(\frac{S_{t_i}}{S_{t_{i-1}}}\right)-K^2\right)^+$$

- Allows to take a position in “volatility-of-volatility”
- No static hedging available
- Need to find a model, where “volatility-of-volatility” can be priced.
- Stochastic volatility models provide such framework.

Model Specification

Time dynamics

- Recall Heston Jump Diffusion Model. Stock dynamics is given by the following:

$$\frac{dS_t}{S_t} = \sqrt{v_t}\left(\rho dW_t - \sqrt{1-\rho^2}\, dW_t^{\perp}\right) + \sum_{i=1}^{N_t} \xi_i - \lambda m t,$$

where jumps are normal with mean μ and volatility σ

- The volatility dynamics is

$$dv_t = \kappa(\theta - v_t)dt + \eta\sqrt{v_t}\, dW_t$$

- For pricing volatility products it is more important to fit variance curve rather than implied volatility surface

Getting variance term structure right

- Fitted Heston approach: making the whole variance curve one of the state variables of the model. In such a model it is easy to compute the Delta with respect to the Variance Swaps.
- For $0 = t_0 < ... < t_n = T$ expected variance at maturity T is defined by

$$V(T) = E\left(\sum_{i=1}^{n} \log^2\left(\frac{S_{t_i}}{S_{t_{i-1}}}\right)\right)$$

- It can be decomposed in jump part and continuous part as following:

$$V(T) = \int_0^T \left\{m_s + \lambda\left(\mu^2 + \sigma^2\right)\right\} ds$$

- So for a given jump parameters we need to fit a continuous part implied from the variance swap market.

Getting variance term structure right

- We can see that it is achieved whenever

$$m_t = E(v_t)$$

- In our “Fitted Heston” model, we redefine v_t, so that

$$v_t^{new} = v_t^{old} \frac{m_t}{E\left(v_t^{old}\right)}.$$

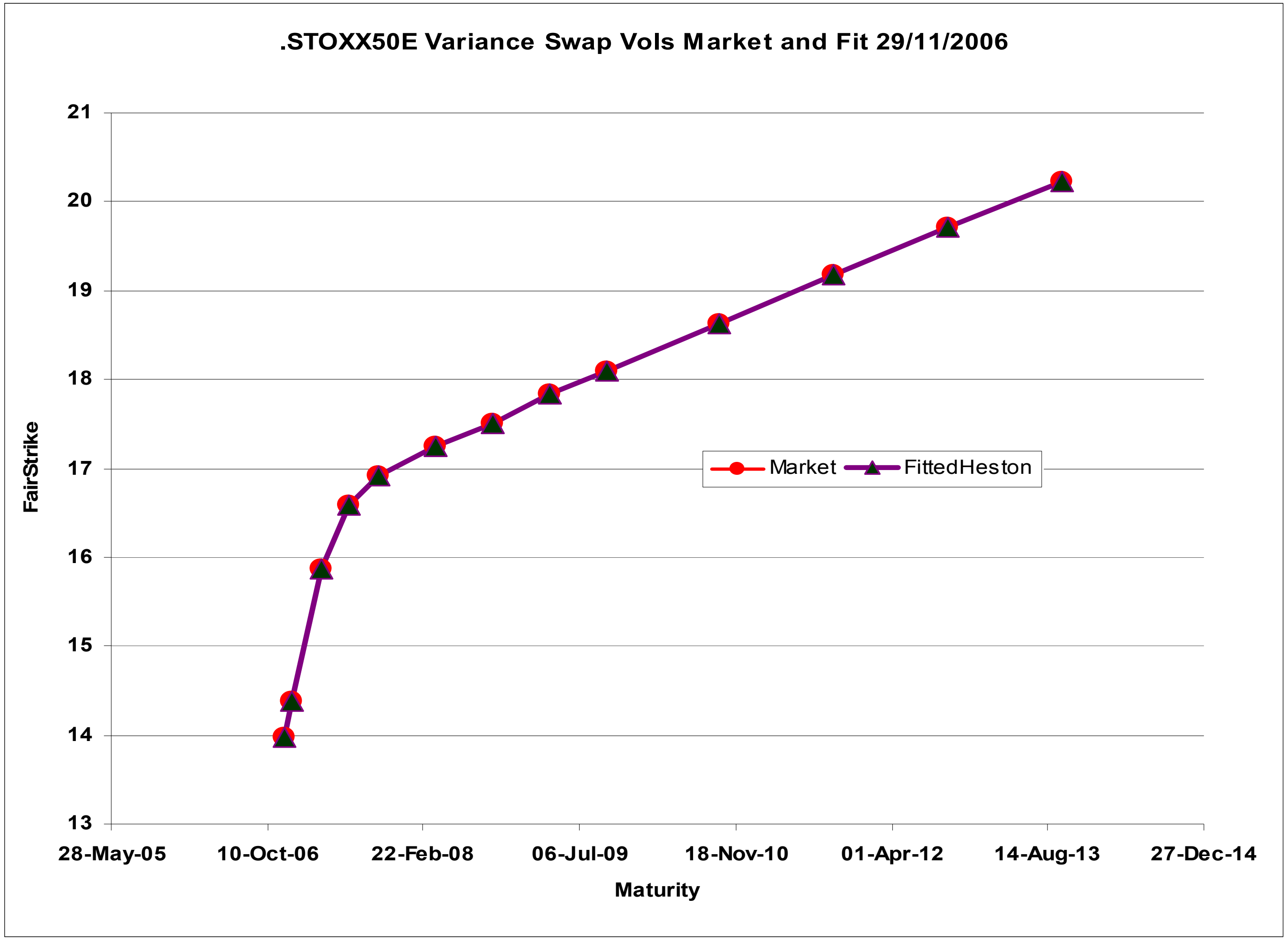
- The resulting model always fits the Variance Swap market (the only error is due to interpolation)

- This approach is similar to the approach in interest rate models, cf. Buehler,H. “Consistent variance curves” *Finance and Stochastics (2006)*

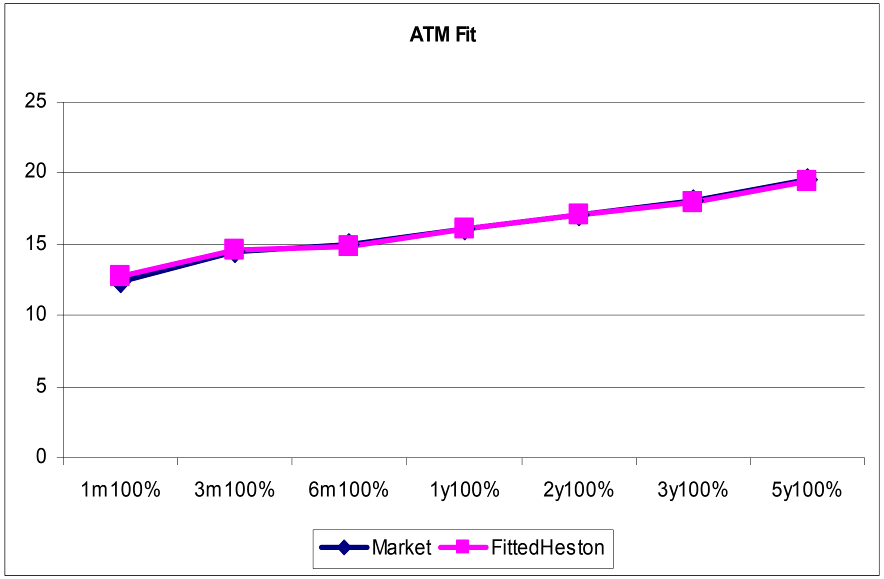
Fitting Fitted Heston

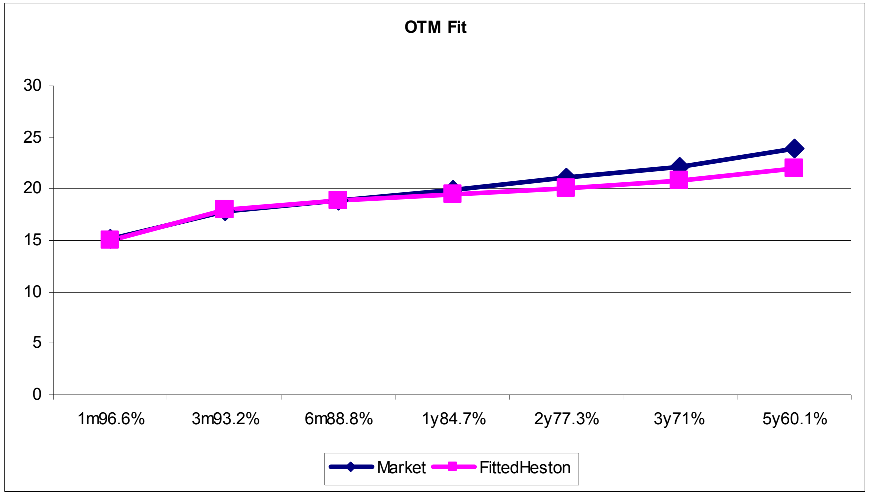
Variance swaps fit

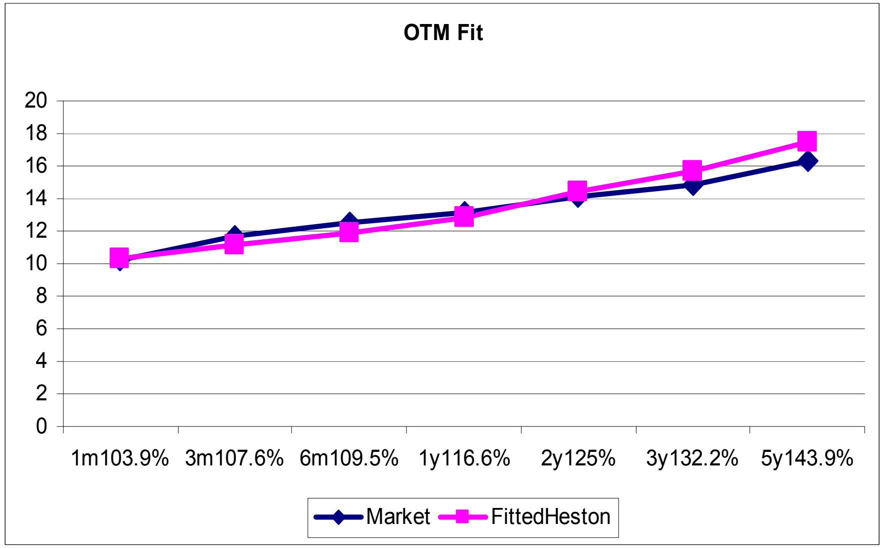
- Variance swap fit. The only error is an interpolation error.



Calibration Fit

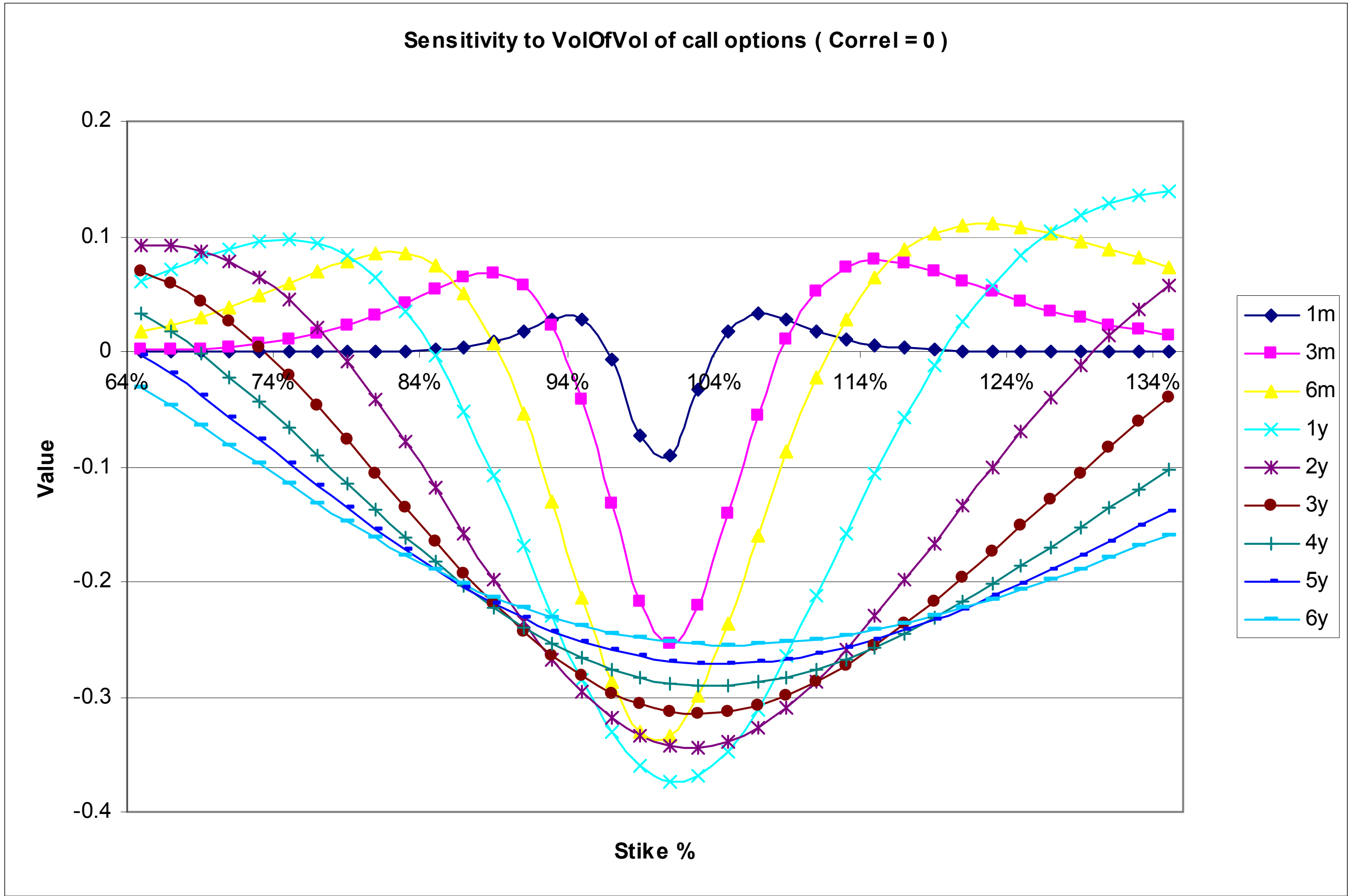




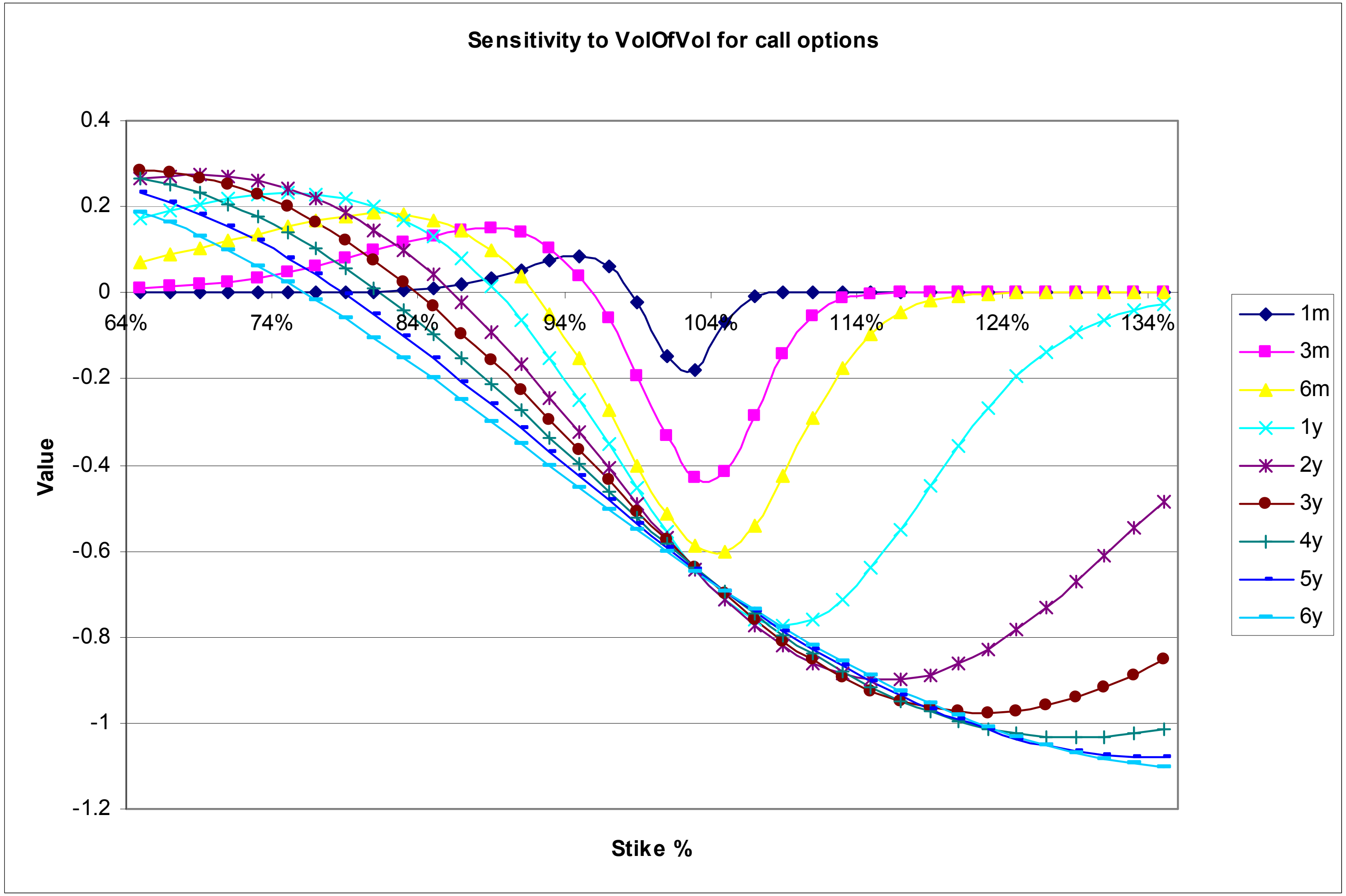


Sensitivities with respect to the parameters of the model

Sensitivity to VolOfVol - correlation = 0 - European Call



Sensitivity to VolOfVol - European Call

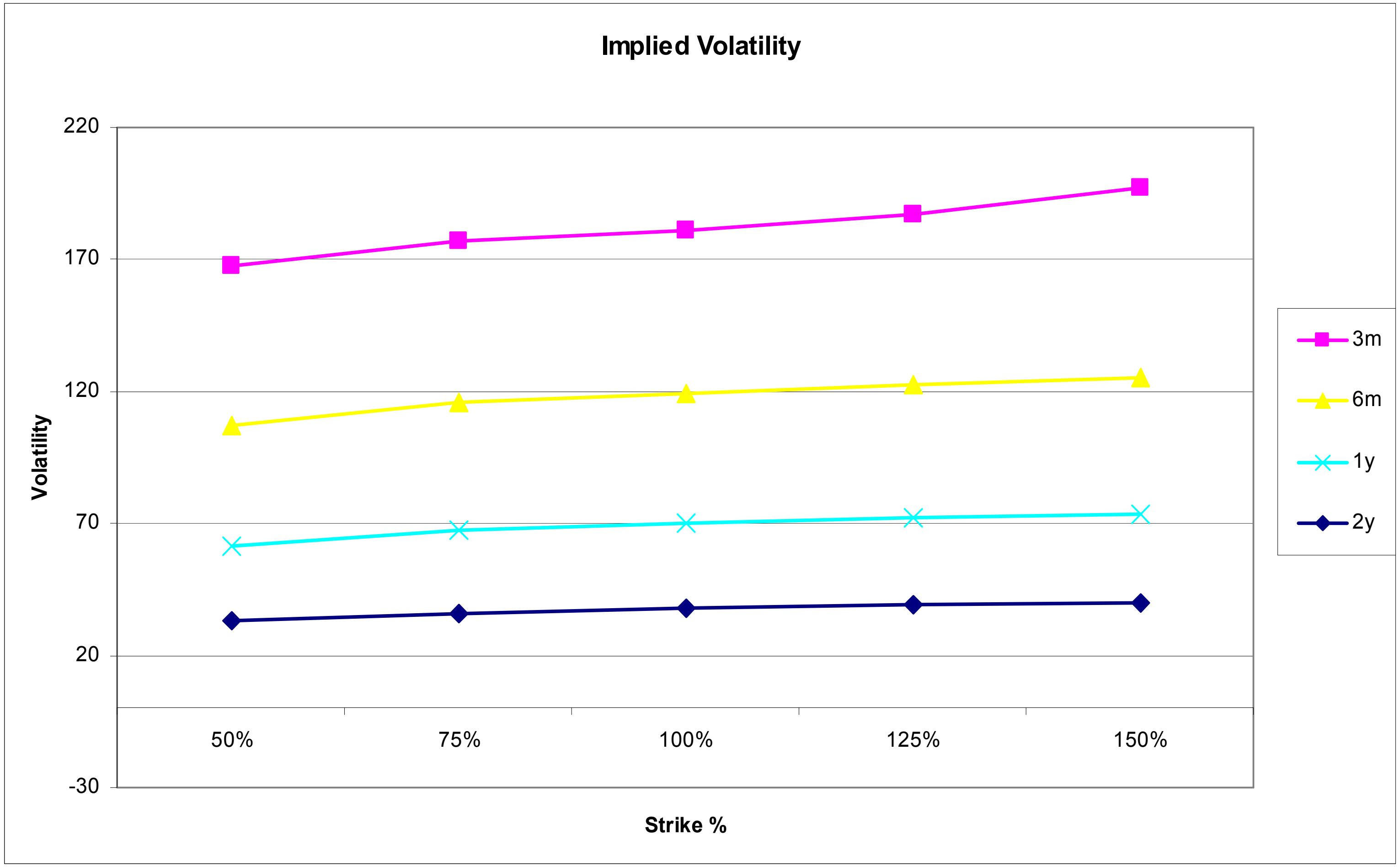


Some conclusions

- In a zero correlation case the sensitivity to the VolOfVol is negative around ATM level and gets positive on the wings. It goes to zero for far out of the money options due to zero vega.
- In the presence of correlation the sensitivities to VolOfVol are negatively skewed.
- The effect of the Reversion Speed on the European Call prices is the inverse to the effect of VolOfVol. Loosely speaking higher Reversion Speed kills faster VolOfVol.

Implied Volatility of Options on Variance

BS Implied Volatility of Call on Variance



BS Implied Volatility of Call on Variance

- Implied Volatility of a Call on Variance is positively skewed due to the effect of the jumps in a Stock process.
- Note that the jumps in the log returns can be negative and positive but contribute only positively to the Realised Variance.
- The skew decreases with the time to maturity.

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